NUMERICAL SOLUTIONS OF ONE-DIMENSIONAL SELF-SIMILAR PROBLEMS OF GAS MOTION IN A POROUS MEDIUM FOR A QUADRATIC DRAG LAW

V. M. Kolobashkin, N. A. Kudryashov,
and V. V. Murzenko

UDC 532.546

Computations are performed of the pressure and velocity of gas motion in a porous medium for self-similar one-dimensional (plane, axisymmetric, and spherically symmetric) problems for a quadratic drag law.

In describing liquid or gas motion in a porous medium, the Darcy law [1] and the empirical Forchheimer equation [2] are used, which for large Reynolds numbers (Re $\geqslant$ 100) can be replaced by a quadratic drag law. The equations for the gas pressure and velocity here allow for self-similar solution for certain initial and boundary conditions, which is of importance since the solutions correspond to actual processes in a certain domain of the variables.

Self-similar solutions of gas filtration problems in a porous medium were considered in [3-8] for a quadratic drag law.

Equations in the self-similar variables were first presented in [4] for gas filtration problems and the asymptotic analysis of the integral curves. An analytic solution of the plane problem is obtained in [5] for a constant pressure on the boundary. The solution of the problem of axisymmetric motion with constant gas mass-flow on the axis is represented in [6]. The numerical solution of the plane problem was examined in [7, 8].

Numerical solutions are presented below for the plane, axisymmetric, and spherically symmetric problems for a zero initial condition and power-law time dependence of the gas mass-flow at the origin.

One-dimensional isothermal gas filtration for a quadratic drag law is described by the following system of equations:

$$\frac{\partial P}{\partial t}+\frac{1}{r^s}\frac{\partial}{\partial r}(r^s uP)=0,\ \frac{\partial P}{\partial r}+\frac{Pu^2}{bc^2}=0, \tag{1}$$

where s = 0 corresponds to the plane problem, s = 1 to the axisymmetric, and s = 2 to the spherically symmetric problem.

We write the initial and boundary conditions for the system (1) in the form

$$P(r, t=0)=0;\ \lim_{r\to 0}(r^s uP)=At^q. \tag{2}$$

The boundary condition corresponds to a liquid or gas mass-flow according to a power law at r = 0.

The problem (1)-(2) is self-similar with the dimensionless variable $\theta = r(bc^2t^2)^{-1/3}$. The system (1) and the conditions (2) are the following in the self-similar variables:

$$mf-\frac{2}{3}\theta f'+\frac{s}{\theta}f\varphi+f'\varphi+\varphi' f=0; \tag{3}$$

$$f'+f\varphi^2=0; \tag{4}$$

$$f(\theta\to\infty)=0,\ \lim_{\theta\to 0}(\theta^s f\varphi)=1. \tag{5}$$

The pressure and velocity of gas motion in a porous medium are expressed in terms of the dimensionless functions $f(\theta)$ and $\varphi(\theta)$:

Moscow Engineering-Physics Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 4, pp. 616-621, October, 1983. Original article submitted May 27, 1982.

0022-0841/83/4504-1155$07.50

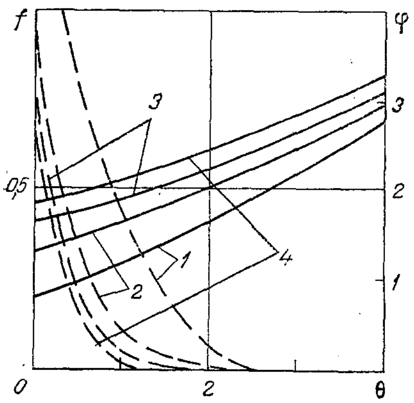

Fig. 1. Dependences of the dimensionless pressures (dashed curves) and velocities of gas motion (solid lines) on θ for q = 0 (curves 1), q = 2 (curves 2), q = 4 (curves 3) and q = 6 (curves 4) for the plane problem (s = 0).

$$P(r;\ t) = \frac{At^m}{(bc^2)^{\frac{1+s}{3}}} f(\theta);\ u(r,\ t) = \left(\frac{bc^2}{t}\right)^{1/3} \varphi(\theta). \tag{6}$$

Substituting (4) into (3) and cancelling $f \neq 0$, we obtain an equation for φ:

$$\varphi' - \varphi^3 + \frac{2}{3}\theta\varphi^2 + \frac{s\varphi}{\theta} + m = 0. \tag{7}$$

The constant m in (6), (7) is related to q and s as $m = q + (1 - 2s)/3$. For s = 0 and m = 0, analytic solutions of (7) are obtained in [5], and they are obtained in [6] for s = 1 and m = 0. In the general case, application of the scheme proposed in [5, 6] for the solution is difficult.

Let us examine the asymptotic behavior of the solutions of (7) for θ >> 1. Analysis of the derivatives φ' on the lines φ = αθ shows that for α > 2/3, φ' → ∞, while for α < 2/3, φ' → −∞. The limit curve to the family of integral curves for any s and m is $\varphi = 2/3\theta + O(\theta^{-2})$. It is shown in [4] that this curve is unique, which satisfies the condition of boundedness of the liquid or gas mass during filtration in a porous medium.

For θ >> 1 the asymptotic solution for φ has the following form to second approximation accuracy:

$$\varphi = \frac{2}{3}\theta + \frac{9}{4\theta^2}\left[m + \frac{2}{3}(1+s)\right] + O(\theta^{-4}). \tag{8}$$

Let us examine the behavior of the solution of (7) for θ << 1. For s = 0 we obtain in (4) and (5) as θ → 0 that φ tends to the constant quantity γ_0, whose value we determine below. For s = 1 and s = 2 we obtain that for θ << 1, φ is related to θ by the dependence

$$\varphi = \sqrt{\frac{2s-1}{2\theta}} + \gamma\theta + O(\theta^{-2}). \tag{9}$$

Let us note that, at first glance, s → 0 should follow from (9) as $\varphi \to \gamma_0$. However, this is not so, since the singularity θ = 0 is eliminated in (1) and (7) as s → 0, and the formal passage mentioned is not realized.

The solution of (7) was carried out by a numerical method by using a four-step explicit Runge–Kutta fourth-order approximation scheme [9]. Because of the necessity to select a specific integral curve satisfying the boundary condition, practical computations are possible only with a negative step. It was assumed that the coordinate θ varied between 10.1 and 0.1 with a step $h = \theta_n - \theta_{n-1} = -0.01$.

The velocity φ_{n-1} (n = N, N−1, ..., 2) in the coordinate θ_{n-1} was defined by the formula

$$\varphi_{n-1} = \varphi_n + h(K_1 + 2K_2 + 2K_3 + K_4)/6, \tag{10}$$

where

$$K_1 = G(\theta_n;\ \varphi_n);\ K_2 = G(\theta_n + 0.5h;\ \varphi_n + 0.5K_1h);$$

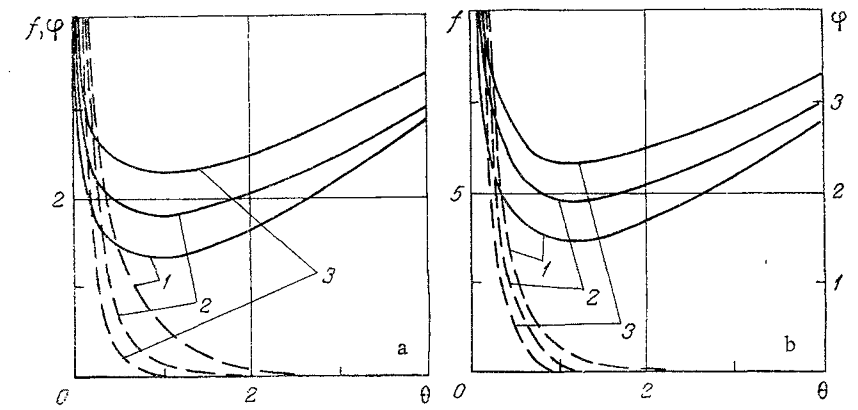

Fig. 2. Dependences of the dimensionless pressure (dashed curves) and velocities of gas motion (solid curves) on the dimensionless variable θ for q = 0 (curves 1), q = 2 (curves 2), and q = 6 (curves 3) for the axisymmetric problem s = 1 (a) and the spherically symmetric problem s = 2 (b).

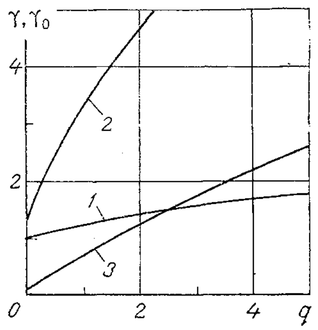

Fig. 3. Dependences of the numerical values of γ_0 and γ in the solutions for the velocity of gas motion on the exponent q in the boundary condition.

$$K_3 = G(\theta_n + 0.5h;\ \varphi_n + 0.5K_2h);\ K_4 = G(\theta_n + h;\ \varphi_n + K_3h);\quad G(x,\ y) = y^3 - \frac{2}{3}xy^2 - \frac{sy}{x} - m. \tag{11}$$

The value of φ_N was calculated from the asymptotic formula (8). The mesh function f_n was also determined numerically in the appropriate coordinates θ_n by using the Runge–Kutta method for the formulas analogous to (10), (11), where $G(x;\ y) = -y\varphi^2(x)$. The value of f_1 was calculated from the boundary condition (5). The computation was later performed with a positive step.

Certain results of the numerical solution of the problem (3)-(5) are represented in Figs. 1-4.

Shown in Fig. 1 are dependences of the pressure f(θ) (dashed curves) and the motion velocity φ(θ) (solid curves) for q = 0, 2, 4, 6, for the plane problem (s = 0). It is seen from the figure that the velocity φ(θ) of gas motion increases as a function of θ. For a fixed time t we obtain from (6) that the values of u(r, t) also increase with distance. This is related to the fact that a jump in the gas pressure exists at the initial time, which indeed yields an infinite value of the velocity. For a fixed coordinate, the velocity values diminish with time.

In reality it is meaningless to speak about gas motion even for $\theta \geqslant 3$, since the gas pressure for such θ is extremely small. Filtration models do not describe the gas motion for the conditions mentioned for small t. Let us note that the increase in velocity with distance is typical for a number of self-similar problems [10, 11].

Shown in Fig. 2 are dependences of the pressure f(θ) (dashed curves) and the velocity of gas motion φ(θ) (solid curves) for q = 0, 2, 6 for the axisymmetric and spherically symmetric problems.

For large θ (r is fixed, t → 0), the solutions for the velocity will be identical for s = 0, 1, 2, as is seen from the figure, which corresponds to the physical crux of the problems under consideration: As t → 0, the characteristic distances of the processes are small and the geometry of the problems exerts no influence. For small θ (r → 0, t, fixed) there are quantitative and qualitative differences in the behavior of the solutions for the velocity of gas motion.

Dependences of γ_0 (curve 1) and γ (curves 2, 3) on the exponent q in the boundary condition are represented in Fig. 3. Numerical computations exhibited good agreement with the asymptotic dependences for $\theta \ll 1$ for problems with different symmetry.

The approximate dependences that agree with the numerical solutions to 5% accuracy for $\theta \leqslant 3$ have the form

$$\varphi \simeq \gamma_0 + 0.25\theta + 0.05\theta^2;\ s = 0;\ q = 0; \tag{12}$$

$$\varphi \simeq \frac{1}{\sqrt{2\theta}} + 0.6\theta;\ s = 1;\ q = 0; \tag{13}$$

$$\varphi \simeq \sqrt{\frac{3}{2\theta}} + 0.37\theta;\ s = 2;\ q = 0. \tag{14}$$

Analogous dependences, but with other numerical values of the coefficients for θ, can be written also for $q \neq 0$. The gas pressure in a porous medium is determined from (4):

$$f = \exp\{-\int \varphi^2 d\theta\}. \tag{15}$$

Substituting (12)-(14) into (15), we can obtain approximate solutions for $f(\theta)$.

It is known [12, 13] that the quadratic drag law is applicable to describe gas motion in a porous medium for Reynolds numbers less than 10^3. This circumstance constrains the applicability of the system (1),(2) in describing gas filtration. The solutions are valid only in a certain domain of the variable θ [5]. For small r and t, the gas motion is described by a gasdynamic system of equations which agrees with the solution of the filtration problem as r and t increase, as the numerical investigation showed.

Results of the present paper can be applied in investigations of gas motion in cracks [14].

NOTATION

P(r, t), pressure; u(r, t), velocity of gas motion; A, dimensional constant; q, exponent; t, time; θ, dimensionless variable; $f(\theta)$, $\varphi(\theta)$, dimensionless pressure and velocity of gas motion; b, coefficient in the quadratic drag; c, speed of sound; α, parameter; $\gamma(q)$, numerical value in the solution; $\gamma_1(\theta)$, dependence in the approximate solution; h, coordinate step; f_n, φ_n, dimensionless mesh pressure and velocity of gas motion; β, parameter.

LITERATURE CITED

1. H. Darcy, Les Fontaines Publiques de la Ville de Dijon, Paris (1856).
2. P. Forchheimer, "Wasserbewegung durch Boren," Zs. des VDI, 45, 1781-1788 (1901).
3. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of Nonstationary Liquid and Gas Filtration [in Russian], Nedra, Moscow (1972).
4. G. I. Barenblatt, "On self-similar compressible fluid motions in a porous medium," Prikl. Mat. Mekh., 16, No. 6, 679-698 (1952).
5. A. G. Bondarenko, V. M. Kolobashkin, and N. A. Kudryashov, "Self-similar solution of the problem of gas flow through a porous medium in the turbulent filtration mode," Prikl. Mat. Mekh., 44, No. 3, 573-577 (1980).
6. N. A. Kudryashov and V. V. Murzenko, "Self-similar solution of the problem of axisymmetric gas motion through a porous medium for a quadratic drag law," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6, 168-171 (1982).
7. F. A. Morrison, Jr., "Transient non-Darcy gas flow in a finite porous bed," Trans. ASME, Ser. I, J. Fluids Eng., 99, No. 4, 779-781 (1977).
8. F. A. Morrison, Jr., "Similarity in transient high-speed gas flow through porous media," Trans. ASME, Ser. I, J. Fluids Eng., 98, 667-671 (1976).
9. S. K. Godunov and V. S. Ryaben'kii, Difference Schemes [in Russian], Nauka, Moscow (1977).
10. L. I. Sedov, Similarity Methods and Dimensional Analysis in Mechanics [in Russian], Nauka, Moscow (1972).
11. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shocks and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1966).

12. P. Ya. Polubarinova-Kochina, Theory of Groundwater Motion [in Russian], Nauka, Moscow (1977).
13. R. I. Nigmatullin, Principles of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
14. J. H. Pitts and H. Brandt, "Gas flow in a permeable earth formation containing a crack," Trans. ASME, Ser. E, J. Appl. Mech., 44, No. 4, 553-558 (1977).

QUESTION OF THE MOVEMENT OF WATER IN CONCRETE WHEN IT FREEZES

F. M. Krantov and A. G. Shlaen UDC 691.32:532.5

We determine the dimensions of capillaries capable of removing the excess water from a freezing pore when there are no destructive processes taking place.

Cement concrete is a capillary–porous solid. In the overall volume of its porosity we generally distinguish two main types of pores and capillaries: cement-gel pores, whose radii vary from $2 \cdot 10^{-9}$ to $2 \cdot 10^{-8}$ m, and capillaries, which have a radius greater than 10^{-7} m [1]. The relation between these types of porosity and the distribution of pores along the radii depends on a number of technological factors and is determined mainly by the composition of the concrete: by the amount of water used and the water-to-cement ratio (w/c). When the concrete freezes, the water in the pores of the gel does not freeze above 233°K [1, 2], but in the capillary pores it freezes at higher temperatures; as a result of the increase in its specific volume, in this phase transition, excess pressures arise in the pore system of the concrete. The stresses in the structure of the concrete which result from these pressures may lead to its failure. The capability of withstanding a specified number of cycles of alternating freezing and thawing while its loss of strength remains within a prescribed limit is called the frost resistance of the water-saturated concrete. The introduction of air-entraining or gas-producing additives into the concrete mix creates in the concrete closed air bubbles with radii of $5 \cdot 10^{-6}$–$2 \cdot 10^{-4}$ m, which are surrounded by the cement gel, do not fill up with water under ordinary conditions, and are connected to the general capillary–porous structure by the pores in the gel [3]. It is known that such bubbles in the concrete help to increase its frost resistance [2, 3]. The greater the number of bubbles and the smaller the distances between them, the greater will be the increase in the frost resistance of the concrete [1-3]. Most investigators – e.g., [1-3] – attribute this to the fact that the air bubbles are compensating volumes into which the excess water can go when the water freezes in the capillaries.

Figure 1 shows a simplified scheme of the structure under consideration. A water-filled cylindrical pore of radius R_p and length l_p is closed at the bottom and surrounded by cement stone containing air bubbles connected with the pore by water-filled capillaries. The connecting capillaries are represented by straight cylindrical channels with an orientation perpendicular to the surface of the filled pore and having an average length and variable radius r_i. In the freezing process the heat is removed from the upper part of the specimen.

If the time required for the freezing of the water in the pore is much longer than the time required to stabilize the freezing rate [4], we can assume that the plane crystallization front moves along the pore axis at constant velocity v_s. The thermal conductivity of the mass surrounding the pore is disregarded, since the supercooling of the water is enough to absorb the heat that is generated. In this case the value of Q, the volumetric flow rate of the water from the water-filled pore, required to prevent an intensive increase in pressure is determined as follows:

"Soyuzvodélektronika" Specialized Design and Technological Office. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 4, pp. 621-625, October, 1983. Original article submitted March 22, 1982.

0022-0841/83/4504-1159$07.50